

**Introduction:** Photometric data for the lunar surface are usually disturbed by resolved topography, especially, at large phase angles. The local surface slopes change the local incidence  $i$  and emergence  $\epsilon$  angles. Therefore, it is impossible to compare the reflectance of flat and topographically disturbed sites. This effect particularly becomes apparent on phase ratio images (see, e.g., [1]). We also illustrate this effect in Fig. 1 that presents a phase ratio image ( $2^\circ/21^\circ$ ) of the lunar disk (see some details below). As one can see, cratered and mountain areas near terminator are not suitable for studying the phase dependence of brightness.

To compensate the influence of the lunar resolved topography on photometric data, it is necessary to use information about local slopes. Unfortunately, there are no suitable digitalized global maps of the lunar relief. However, if photometric data on the same areas are obtained at different angles  $i$  and  $\epsilon$ , one can retrieve information about the relief and albedo distribution from the data. We demonstrate this possibility with Earth-based observations of the Moon using a small telescope and a Canon EOS 300D camera.

**Retrieving lunar topography from photometric data:** The idea of the technique we used is based on our early work [2]. Let us define the local relief slope  $r$  as the declination of the surface normal from the global vertical. The resolution element of our data is about  $2'' \times 2''$  or approximately  $4 \times 4 \text{ km}^2$  on the lunar surface. This just is the minimum roughness size we deal with.

The local slopes of the surface affect the local illumination/observation geometry and, hence, the photometric coordinates  $\lambda$  and  $\beta$  of a given point (the definition of photometric longitude  $\lambda$  and latitude  $\beta$  see, e.g., in [3]). The slope disturbing of local sites is equivalent to the displacement of the sites to the point with new photometric coordinates  $\lambda' = \lambda + \Delta\lambda$  and  $\beta' = \beta + \Delta\beta$  – longitude and latitude, respectively. Denote the longitudinal and latitudinal components of the local slope  $r$  as  $r_l$  and  $r_b$ :

$$r_l = r \sin\varphi, \quad r_b = r \cos\varphi,$$

where  $\varphi$  is the azimuth angle of the projection of local normal on the average plane, which is measured from the direction to the north pole. The quantities  $r_l$  and  $r_b$  can be expressed through  $\Delta\lambda$  and  $\Delta\beta$ .

In contrast with [2], where we used a linear approach, assuming that the slopes are small, in this work we develop a new algorithm without any restrictions on the character of the dependence of the disk function on the photometric coordinates. The

gist of the new approach is to minimize the standard deviation of the equigonal albedo (that is by the definition:  $A'_{eq}(\alpha) = A(\alpha, \lambda', \beta') / \psi(\alpha, \lambda', \beta')$  [4]) for each point from values calculated using the model phase function:  $A_{eq}(\alpha) = m \exp(-\mu\alpha)$ , where  $m$  is a normalizing coefficient,  $\mu$  is the parameter that characterizes the steepness of the phase curve,  $A(\alpha, \lambda', \beta')$  is the observed albedo,  $\psi(\alpha, \lambda', \beta')$  is the disk function, and  $\alpha$  is the phase angle [4].

For the minimization we use the Nelder-Mead method [5]. The parameters  $r_l$ ,  $r_b$ ,  $m$ , and  $\mu$  are fitted, using a set of observations of the Moon carried out at various phase angles before and after full-moon. Thus, using this algorithm we map simultaneously the topographical slopes and the parameters of brightness phase dependence not distorted by relief.

As a disk function we used the factorized expression  $\psi(\alpha, \lambda, \beta) = \psi_\lambda(\alpha, \lambda) \psi_\beta(\alpha, \beta)$ . Usually the Lommel-Zeeliger law is used as a disk function, e.g., [3]. For this case  $\psi_\beta(\alpha, \beta) = 1$  and

$$\psi_\lambda(\alpha, \lambda) = \frac{\cos(\alpha - \lambda)}{\cos \lambda - \cos(\alpha - \lambda)}$$

However, there are much more exact disk functions suggested by Akimov [4]. We use the following Akimov's components of the disk function

$$\psi_\lambda(\alpha, \lambda) = \cos \frac{\alpha}{2} \cos \left[ \frac{\pi}{\pi - \alpha} \left( \lambda - \frac{\alpha}{2} \right) \right] / \cos \lambda$$

and

$$\psi_\beta(\alpha, \beta) = (\cos \beta)^{q(\alpha)}$$

where  $q(\alpha)$  is the so-called roughness factor [6,7]

$$q(\alpha) = \frac{v\alpha}{\pi - \alpha}$$

where  $v = 0.34$  for maria and  $v = 0.52$  for highlands. We used the mean value  $v = 0.43$ .

The relations between the selenographic and photometric coordinates are given in [8].

**Results of calculations:** For calculations we used the data of absolute photometry of the Moon carried out in 2006 at the Maidanak observatory (Uzbekistan) with a 15-cm refractor at red light 610 nm [9]. Data-set of 22 maps of absolute albedo [9] obtained at  $\alpha = 12.2^\circ \dots 21.0^\circ$  before full-moon and  $\alpha = 12.3^\circ \dots 21.7^\circ$  after full-moon. Observation data were selected to have maximally symmetrical values of phase angles. Range of phase angles from  $12^\circ$  to  $22^\circ$  allows us to use a relatively simple model phase function,  $A_{eq}(\alpha) = m \exp(-\mu\alpha)$ . The maximal phase angle  $22^\circ$  allows us to construct maps for almost full visible disk of the Moon in contrast to our previous work [2].

Unfortunately, it is difficult to retrieve latitudinal

component  $r_b$  from Earth-based observations, because of  $\psi_\beta$  of a given point varies slightly (mainly due to libration) in contrast to  $\psi_\lambda$ . Therefore, we made the optimization procedure only for 3 parameters  $r_l$ ,  $m$ , and  $\mu$ . In Fig. 2 a map of slopes along the selenographic longitude  $r_l$  on the base 4 km is shown. The distribution of the longitudinal slopes over the lunar disk shows realistic values, up to  $8^\circ$ . Some sites demonstrate slopes reaching  $20^\circ$ , however, we say about it with precaution, as those could be related to errors as well. This new map in general coincides with our previous results of mapping [2].

**Topography correction of photometric data:**

Although we mapped the longitudinal component only, nevertheless this allows correction of photometric observations, because lunar illumination varies mainly due to changing selenographic longitude of the Sun. For the correction of maps of albedo, it is necessary to apply the following correcting coefficient for each point of the lunar disk

$$k = \psi(\alpha, \lambda, \beta) / \psi(\alpha, \lambda', \beta'),$$

where  $\psi(\alpha, \lambda, \beta)$  and  $\psi(\alpha, \lambda', \beta')$  are the disk functions, respectively, without and with taking into account topography slopes.

In Fig. 3 a corrected map of phase ratio  $2^\circ/21^\circ$  is presented. For this map the influence of topography is significantly suppressed (cf. Fig.1). As can be seen, the phase ratio images reveal a correlation with albedo. However, detailed comparison shows that there are many anomalous regions that do not exactly coincide with albedo boundary. For instance the Aristarchus Plateau and Marius Hills (pyroclastic formations) clearly show up on the phase ratio images. We note that in the latter case the topography correction help to see better the anomaly.

**Conclusions:** (1) A new method to map the longitudinal slopes of lunar topography using photometric observations has been developed. (2) The obtained map allows us to correct photometric data for the topography influence.

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**References:** [1] Shkuratov Yu., et al. (1994) *Icarus*, 109, 168-190. [2] Korokhin V., Akimov L. (1997) *Solar System Res.*, 31, 143-152. [3] Hapke B., (1993) *Theory of reflectance and emittance spectroscopy*, Cambridge Univ. Press, 450 p. [4] Akimov L. (1988) *Kinematics Phys. Celest. Bodies*, 4, 1, 3-10; and 2, 10-16. [5] Nelder J., Mead R. (1965) *Computer Journal*, 7, 308-313. [6] Akimov L., et al. (1999) *Kinematics and Phys. of Celest. Bodies*, 15, 4, 304-309. [7] Akimov L., et al. (2000) *Kinematics Phys. Celest. Bodies*, 16, 2, 181-187. [8] Korokhin V., Akimov L. (1994) *Kinematics Phys. Celest. Bodies*, 10, 2, 3-10. [9] Velikodsky Yu. et al. (2009) this issue.

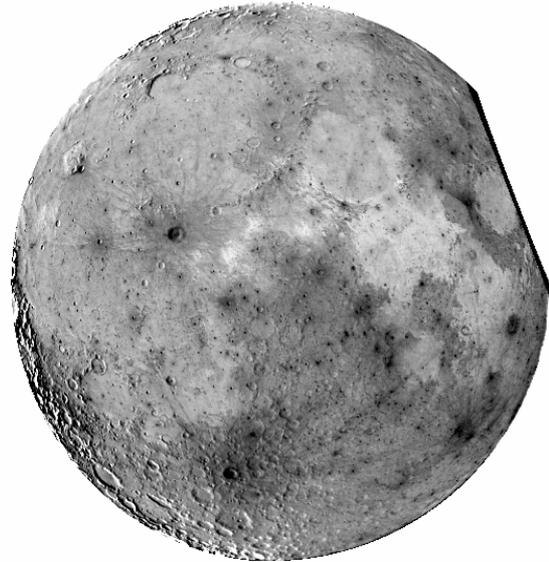


Fig. 1. Phase ratio  $2^\circ/21^\circ$  without topography corrections

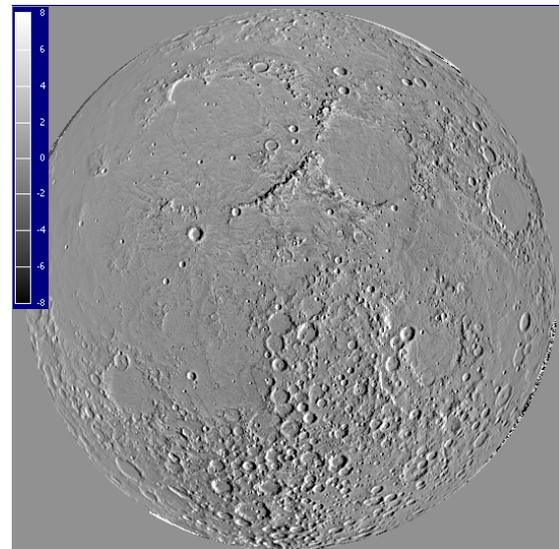


Fig. 2. Map of slopes along selenographic longitude on the base 4 km. Scale in degrees.

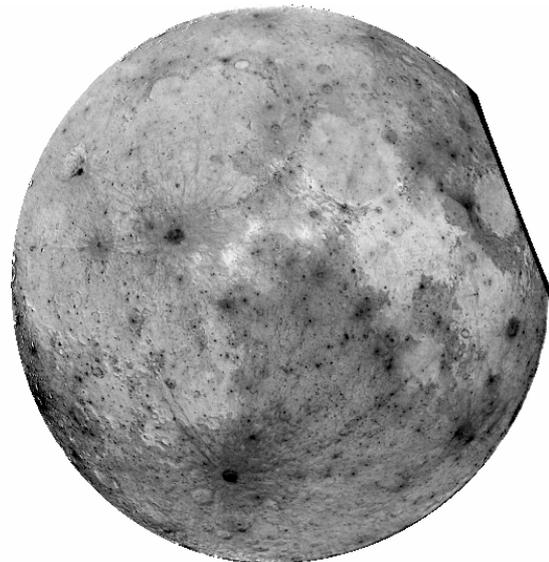


Fig. 3. Topographically corrected phase ratio  $2^\circ/21^\circ$ .