EFFECTS OF SECOND APPROXIMATION FOR FIGURES AND GRAVITATIONAL MOMENTS OF JUPITER'S SATELLITES IO AND EUROPA. V. N. Zharkov, Schmidt Institute of Physics of the Earth, Russian Academy of sciences, 123495 Moscow, B.Gruzinskaya, 10, zharkov@ifz.ru

The gravitational moments (J_2 and C_{22}) of Io and Europa were first determined in [1, 2] as a result of the successful Galileo space mission. These data were refined in the review [3] and are given in Table 1 (the data are used as boundary condition). Two types of trial three-layer models have been constructed for the satellites Io and Europa. In the models of the first (type Io1 and E1), the cores are assumed to consists of the eutectic Fe-FeS melt with the densities ρ_1=5.15 g cm^{-3} (Io1) and 5.2 g cm^{-3} (E1). In the models of the second type (Io3 and E3), the cores consist of FeS with an admixture of nickel and have the density ρ_1=4.6 g cm^{-3}. The approach used here differs from that used previously both in chosen model chemical composition of these satellites and in boundary conditions imposed on the models. The parameters of the models are collected in Table 2 and Table 3. The theory set out in [4] allows all three principal, nondimensional moments of inertia normalized to ms_1^2 to be calculated for the constructed models (see Table 2 and Table 3).The most important question to be answered by modeling the internal structure of the Galilean satellites is that of the condensate composition at the formation epoch of Jupiter's system. In first approximation among the satellite's gravitational moments of the second degree and the second order, only J_2 and C_{22} are nonzero, while $C_{22} = S_{21} = S_{22} = 0.$

$$J_2 = \frac{C - 1/2(A+B)}{ms_1^2} = \frac{5}{6}\alpha k_2 \qquad (1)$$

$$C_{22} = \frac{B-A}{4ms_1^2} = \frac{1}{4}\alpha k_2 , \quad C_{22} = \frac{3}{10} J_2 \qquad (2)$$

$$\alpha = \frac{M}{m}\left(\frac{s_1}{R}\right)^3 = \frac{\omega^2 s_1^3}{Gm} \qquad (3)$$

Here, α is the small parameter of the figure theory, M and m are the masses of Jupiter and the satellite, respectively, G is the gravitational constant, s_1 is the mean radius of the satellite; C is the polar moment of inertia; A and B are the equatorial principal moments of inertia; k_2 is the Love tidal number of the second degree; and R is the orbital radius. The number k_2 or h_2=1+ k_2 that is used as the constraint in modeling the Galilean satellite can be determined from formula (2), which is derived in theory to the terms of the first order in small parameter α (3). In theory, we have the terms of the second order in α [4]

$$C_{22} = \frac{1}{4}k_2\alpha - \frac{16}{21}(1 - \frac{5}{16}h_2)h_2\alpha^2 \qquad (4)$$

$$\frac{J_2}{C_{22}} = \frac{10}{3}[1 + \frac{16}{21}\frac{h_2}{k_2}(3 - h_2)\alpha] \qquad (5)$$

The third spherical function W_3 in the tidal potential is proportional to $\alpha_1 \equiv \alpha(s_1/R)$. For Io, the ratio $s_1/R \approx 432 \times 10^{-5}$. Consequently, α_1 is of the order of α^2. If satellite is in state of hydrostatical equilibrium, then all odd gravitational moments

$$S_{3m} = 0 , \quad (m=1, 2, 3) \qquad (6)$$

and only

$$C_{31} = -\frac{1}{4}\alpha_1 k_3 = -\frac{1}{4}\alpha_1(h_3 - 1) ,$$

and

$$C_{33} = \frac{1}{24}\alpha_1 k_3 = \frac{1}{24}\alpha_1(h_3 - 1) \qquad (7)$$

Table 1. Observational data and model parameters for Io and Europa (a - orbital radius, τ - period)

Parameter	Io, J1	Europa,J2
a , 10^3km	421.6	670.9
a/R_J	6.0275	9.592
τ , days	1.769	3.551
r_0, km	1821.6 ± 0.5	1565.0 ± 8.0
m, 10^{23} g	893.19~893.2	480.0
ρ_0, g cm^{-3}	3.5275 ± 0.0029	2.989 ± 0.046
q_0, cm s^{-2}	179	131
α, 10^{-5}	171.37	50.19
v_{2k} ,km s^{-1}	2.55	2.02
J_2 , 10^{-6}	1859.5 ± 2.7	435.5 ± 8.2
C_{22}, 10^{-6}	558.8 ± 0.8	131.5 ± 2.5
C/ms_1^2	0.37824 ± 0.0002	0.346 ± 0.005
Gm, km^3c^{-2}	2	3202.72 ± 0.0
	5959.91 ± 0.02	2
k_2	1.3043 ± 0.0019	1.048 ± 0.020

Table 2. Parameters of the three-layer models for Io

Parameter	Io1	Io3
s_1 ,km	1821.6	1821.6
ρ_0, g cm^{-3}	3.5275	3.5275
Core density ρ_1, g cm^3	5.15	4.6
Mantle density ρ_2, g	3.35	3.32
cm^3	2.7	2.7
Crust density ρ_3, g cm^3	903.16	1053.29
Core radius s_c , km	1781.6	1781.6
Mantle radius s_m , km	1.3032	1.3053
k_2	17.8	25.2
Core mass m_c , wt %	0.376877	0.377097
$\overline{A}$	4	7
$\overline{B}$	0.378366	0.378589
$\overline{C}$	7	0.379707
h_3	0.379483	4
	6	1.5946
	1.59777	

Table 3. Parameters of the three-layer models for Europa

Parameter	E1	E3
s_1 , km	1565.0	1565.0
ρ_0, g cm^{-3}	2.989	2.989
Core density ρ_1, g cm^3	5.2	4.6
Mantle density ρ_2, g	3.31	3.30
cm^3	1.05	1.05
Crust density ρ_3, g cm^3	701.5	806.7
Core radius s_c ,km	1442.23	1442.23
Mantle radius s_m , km	1.043767	1.050487
k_2	17	5
Core mass m_c , wt %	0.345767	22.82
$\overline{A}$	7	0.346659
$\overline{B}$	0.346146	2
$\overline{C}$	0.346407	0.347010
	6	7
		0.347274
		3

are nonzero

$$J_3 = 0 \quad \text{and} \quad C_{32} = 0 \qquad (8)$$

In the first nonvanishing approximation of the theory of figure, we have relation (2) that was used to judge whether the Galilean satellites of Jupiter have equilibrium figures. The same relation for the third spherical function is

$$C_{31} / C_{33} = -6 \;. \qquad (9)$$

If it turns out that relations (9) and (8) hold for Io, then this will imply that Io is in a more "detailed" hydrostatic equilibrium than can be judged from the fulfillment of relation (2) alone. Values of number h_3 may be found in Table 2. For Europa, the considered effect is approximately a factor of 3 smaller, which roughly corresponds to a ratio of the small parameters for the satellites under consideration, $\alpha_{Io} / \alpha_{Europa}$~3.4. Our theory allows the parameters of the figure (s_{nm}) and the forth-order gravitational moments that differ from zero to be calculated [4]. For the homogeneous model, their values are:

$$s_4 = \frac{885}{224}\alpha^2, \qquad s_{42} = -\frac{75}{224}\alpha^2,$$

$$s_{44} = \frac{15}{896}\alpha^2, \qquad J_4 = -\frac{885}{224}\alpha^2,$$

$$C_{42} = -\frac{75}{224}\alpha^2, \; C_{44} = \frac{15}{896}\alpha^2 .$$

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